CYCLES & VARS.

This Lecture is based Blanchard's lecture :

http://ocw.mit.edu/courses/economics/14-452-macroeconomic-theory-ii-spring-2007/lecture-notes/slides01.pdf

It forms part of the masters course at MIT in the USA.

At an important part of applied macroeconomics (as opposed to theoretical macroeconometrics) examines the following sort of issues:

(i) How long do booms/recessions last?

(ii) How do Consumption, and Investment move with output?

(iii) How does nominal/real money stock move with output?

(iv) How do nominal/real interest rates move with output?

(v) How do real wages move with output?

But before we can look for patterns in data over time we need to assume/establish that the variables are "covariance stationarity". What do we mean by this?

Let Y_t be a random variable. Then, Y_t is ***covariance stationary*** [more often we just say stationary] iff:

$E[Y_t] = \mu$ for all t - the variable has a mean which does not change over time, e.g. inflation yes possibly stationary as it has a mean, the consumer price index no, it has no mean which is constant over time. Note E[.] denotes the expected value of what lies inside [.]. Think of it as an average value. The subscript t denotes the time period.

$E[(Y_t - \mu)(Y_{t-k} - \mu)] = g_k$ for all t. The covariance between two values of Y k periods apart also does not change. Actually tests of stationarity tend to revolve around the following. Estimate the relationship: $Y_t = \rho Y_{t-1}$

If $\rho<1$ [tend not to consider negative values] we say its stationary. That's not exactly the test, but to do more takes us into econometrics.

If a variable(s) is stationary, then can actually learn/estimate the variance, the stochastic process [what drives the data], the cross-correlations [between one variable over time, or between two different variables]. If it is not stationary, then it becomes more complex to analyse.

Is covariance stationarity a reasonable assumption? Sometimes not. Great Depression. Hyperinflations. Transition in Eastern Europe. Emerging market economies: Sudden stops. The figure on slide 9 from the MIT presentation suggests there have been occasional periods when it is unlikely that what we see comes from a stationary process [i.e. the variable is (covariance) stationary, note we generally omit the word covariance and just refer to stationarity. It is a critical concept in modern macroeconomics].

NOW IF a series can be established as stationary. Then the Wold decomposition theorem becomes valid. This says that the variable can be represented as *an infinite moving average representation*. Big words, but simple really; the series can be represented as an infinite average of current and past random error terms [what economists often call 'shocks']. That's great, but how to estimate an *infinite* series? Well, *infinite* MAs obviously cannot be estimated, But they can be approximated quite well by an AR(n) process. For example an AR(2) [autoregressive process of order 2]:

AR(2) because the lag goes back 2 periods

$Y_t = \rho_1 Y_{it-1} + \rho_2 Y_{2t-2} + \varepsilon_t$ (1)

Similarly if we have say three variables (all stationary, e.g. inflation growth and (hopefully stationary) unemployment). Then we can estimate the system as a VAR (vector auto regression):

$y_t = A1 y_{t-1} + A2 y_{t-2} + \dots\dots\dots + \varepsilon_t$ (2)

where

$$A1 = \begin{bmatrix} a^1_{11} & a^1_{12} & a^1_{13} \\ a^1_{21} & a^1_{22} & a^1_{23} \\ a^1_{31} & a^1_{32} & a^1_{33} \end{bmatrix} \quad y_t = \begin{bmatrix} y1t \\ y2t \\ y3t \end{bmatrix} \; and \; A2 = \begin{bmatrix} a^2_{11} & a^2_{12} & a^2_{13} \\ a^2_{21} & a^2_{22} & a^2_{23} \\ a^2_{31} & a^2_{32} & a^2_{33} \end{bmatrix} \text{ and } \varepsilon_t = \begin{bmatrix} \varepsilon 1t \\ \varepsilon 2t \\ \varepsilon 3t \end{bmatrix}$$

Inserting into (2) we get:

$$\begin{bmatrix} y1t \\ y2t \\ y3t \end{bmatrix} = \begin{bmatrix} a^1_{11} & a^1_{12} & a^1_{13} \\ a^1_{21} & a^1_{22} & a^1_{23} \\ a^1_{31} & a^1_{32} & a^1_{33} \end{bmatrix} \begin{bmatrix} y1t-1 \\ y2t-1 \\ y3t-1 \end{bmatrix} + \begin{bmatrix} a^2_{11} & a^2_{12} & a^2_{13} \\ a^2_{21} & a^2_{22} & a^2_{23} \\ a^2_{31} & a^2_{32} & a^2_{33} \end{bmatrix} \begin{bmatrix} y1t-2 \\ y2t-2 \\ y3t-2 \end{bmatrix} + \begin{bmatrix} \varepsilon 1t \\ \varepsilon 2t \\ \varepsilon 3t \end{bmatrix}$$

Reading across the first row:

$Y_{1t} = a^1_{11} y_{1t-1} + a^1_{12} y_{2t-1} + a^1_{13} y_{3t-1} + a^2_{11} y_{1t-2} + a^2_{12} y_{2t-2} + a^2_{13} y_{3t-2} + \varepsilon_{1t}$ (3)

and similarly for the other two rows. Equation is fairly simple really. It links the current value of y_1 to values of all three variables in t-1 (the previous period, say last year) and all three variables in t-2 (say two years ago). ε_t ? That is simply a random error term, which exists because the lagged values of these three variables are not going to fully explain y_{1t}. Similarly for the other two variables, e.g.:

$Y_{2t} = a^1_{21} y_{1t-1} + a^1_{22} y_{2t-1} + a^1_{23} y_{3t-1} + a^2_{21} y_{1t-2} + a^2_{22} y_{2t-2} + a^2_{23} y_{3t-2} + \varepsilon_{2t}$ (3)

So estimating a VAR is pretty simple really. First establish the variables are stationary. Next do the estimations. But why would it be valid? What would it tell us? Well before we answer this, consider the following system of equations. This is a **structural relationship** linking current values of y_1 to current values of y_2 and y_3. y_1 could be inflation, y_2 unemployment and y_3 money supply growth (hopefully all are stationary). Economic theory underlies these relationships.

$Y_{1t} = c_{12}y_{2t} + c_{13}y_{3t} + a_{11}y_{1t-1} + a_{12}y_{2t-1} + a_{13}y_{3t-1} + u_{1t}$ (4i)

$Y_{2t} = c_{21}y_{1t} + c_{23}y_{3t} + a_{21}y_{1t-1} + a_{22}y_{2t-1} + a_{23}y_{3t-1} + u_{2t}$ (4ii)

$Y_{3t} = c_{31}y_{1t} + c_{32}y_{2t} + a_{31}y_{1t-1} + a_{32}y_{2t-1} + a_{33}y_{3t-1} + u_{3t}$ (4iii)

Now we can rewrite this as:

$y_t = Cy_t + Ay_{t-1} + u_t$ (5)

which can then be rewritten as

$Iy_t = Cy_t + Ay_{t-1} + u_t$ (6)

where I is the identity matrix. The key property of an identity matrix is that when it multiplies another matrix/vector it leaves it unchanged [i.e. $Iy_t = y_t$]. What does it look like? In this case:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

OK now rearrange (6):

$Iy_t - Cy_t = Ay_{t-1} + u_t$ (7)

$[I-C]y_t = Ay_{t-1} + u_t$ (8)

Just one more step

$[I-C]^{-1}[I-C]y_t = [I-C]^{-1}Ay_{t-1} + [I-C]^{-1}u_t$ (9)

$[I-C]^{-1}$ is called the inverse matrix of [I-C]. Just as 1/6 is the inverse of 6. Multiply the two together you get 1; similarly $[I-C]^{-1}[I-C]=I$. This leaves us with:

$y_t = [I-C]^{-1}Ay_{t-1} + [I-C]^{-1}u_t$ (7)

Now those of you who know matrix algebra will find this easy. The rest will find it difficult. The key point is that any structural relationship linking variables to current and past values as in (5) can be transformed into a VAR. **AND VICE VERSA**. Having estimated a VAR we should be able to obtain estimates of the underlying structural equations and the underlying error terms or shocks [the u_t's]. But in there we have a problem, for a VAR can be traced back to several different structural relationships. In order to proceed further to get to a unique structural relationship we need to provide more information in the form of 'identification restrictions'. These could for example be that c_{31} in 4iii is zero, i.e. y_{1t} has no impact on y_{3t}. Alternatively it may be that the long term impact of a shock in y_1 on y_3 is zero. The shock in y_1 is represented by u_{1t} the error term in the structural equation. Having estimated a structural VAR it is common place to estimate the impact of a shock in say y_1 in period t [say inflation] on both inflation itself and the other variables. We will be looking at this in another lecture.

Below we present a very simply VAR. It estimates the following equations:

$D\_lrgrossinv_t = 0.278\ D\_lrgrossinv_{t-1} + 0.576\ D\_lrconsump_{t-1} + 0.00071$

$D\_lrconsump_t = 0.0661\ D\_lrgrossinv_{t-1} + 0.0613\ D\_lrconsump_{t-1} + 0.00772$

Now D_.. stands for the change in or the first difference. Lrgrossinv is the log of real gross investment. Put both together and the change in the log of real gross investment is simply its growth rate. The other variable is the growth of real consumption.

```
. varbasic D.lrgrossinv D.lrconsump if tin(,2005q4), lags(1) irf

Vector autoregression

Sample:  1959q3 - 2005q4                        No. of obs      =        186
Log likelihood =  1191.069                      AIC             = -12.74267
FPE            =  1.00e-08                      HQIC            = -12.70051
Det(Sigma_ml)  =  9.40e-09                      SBIC            = -12.63862

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
D_lrgrossinv          3     .017469   0.1838   41.88835   0.0000
D_lrconsump           3     .006667   0.0516   10.12045   0.0063
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_lrgrossinv |
  lrgrossinv |
         LD. |   .2784748   .0803322     3.47   0.001     .1210267    .4359229
             |
   lrconsump |
         LD. |   .5756131   .2262345     2.54   0.011     .1322017    1.019025
             |
       _cons |   .0007103   .0021455     0.33   0.741    -.0034948    .0049155
-------------+----------------------------------------------------------------
D_lrconsump  |
  lrgrossinv |
         LD. |   .0660611   .0306599     2.15   0.031     .0059688    .1261534
             |
   lrconsump |
         LD. |    .061333   .0863456     0.71   0.478    -.1079013    .2305674
             |
       _cons |   .0077298   .0008189     9.44   0.000     .0061248    .0093348
------------------------------------------------------------------------------
```

Impulse Response Functions

These trace the impact of shocks to the system. Let us assume that in period τ There is a one unit shock to investment. How will the system respond? Well in τ+1 we have

$D\_lrgrossinv_{\tau+1} = 0.278 \times 1 + 0.576 \times 0 = 0.278$

$D\_lrconsump_{\tau+1} = 0.0661 \times 1 + 0.0613 \times 0 = 0.061$

Going forward we have:

$D\_lrgrossinv_{\tau+2} = 0.278\ D\_lrgrossinv_{\tau+1} + 0.576\ D\_lrconsump_{\tau+1}$

$D\_lrconsump_{\tau+2} = 0.0661\ D\_lrgrossinv_{\tau+1} + 0.0613\ D\_lrconsump_{\tau+1}$

OR

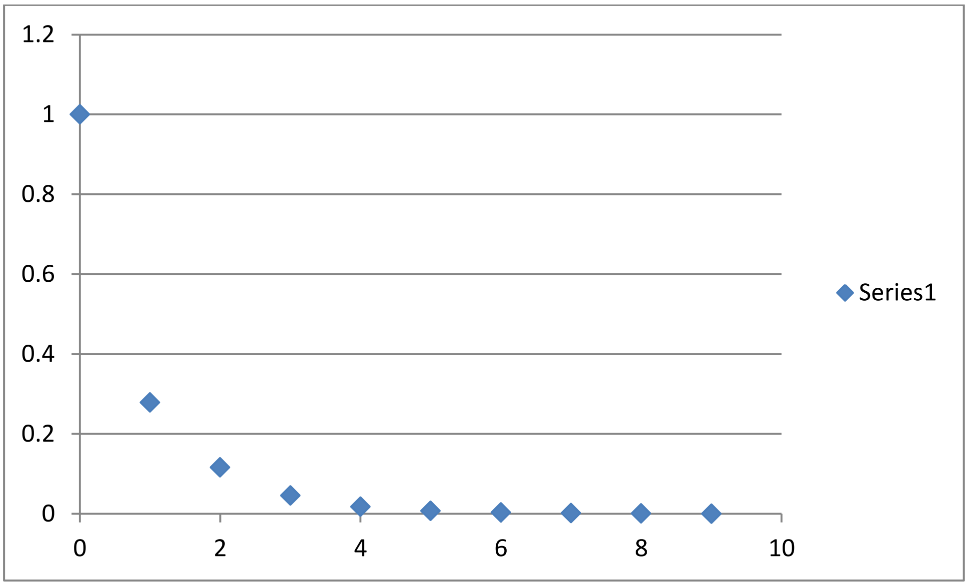
$D\_lrgrossinv_{\tau+2} = 0.278\ 0.278 + 0.576\ 0.061$

$D\_lrconsump_{\tau+2} = 0.0661\ 0.278 + 0.0613\ 0.061$

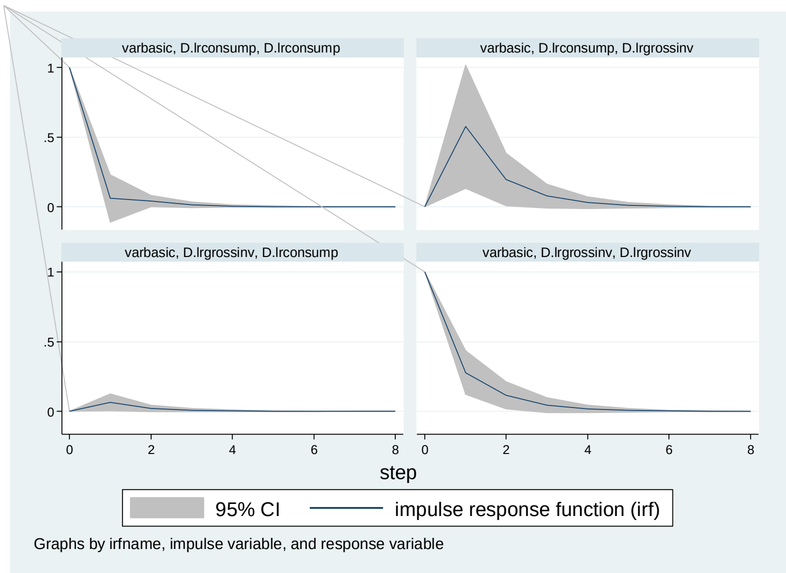
And so on as we move further forward [Note we are missing out the constant terms as these are there regardless of whether there is a shock or not.]. These are done in the Excel file:

	Investment equation	Consumption Equation		Investment equation	Consumption Equation
Shocks:	1	0	Shocks:	0	1
1	0.2784748	0.0660611	1	0.575613	0.061333
2	0.115573849	0.022448077	2	0.195598	0.041787
3	0.045105812	0.009011743	3	0.078522	0.015484
4	0.017748109	0.003532457	4	0.03078	0.006137
5	0.00697573	0.001389116	5	0.012104	0.00241
6	0.002742158	0.000546023	6	0.004758	0.000947
7	0.00107792	0.000214639	7	0.00187	0.000372
8	0.000423723	8.4373E-05	8	0.000735	0.000146
9	0.000166562	3.31664E-05	9	0.000289	5.75E-05

And plotted for the impact of an investment shock on investment:



Most econometric package programs will plot these out.

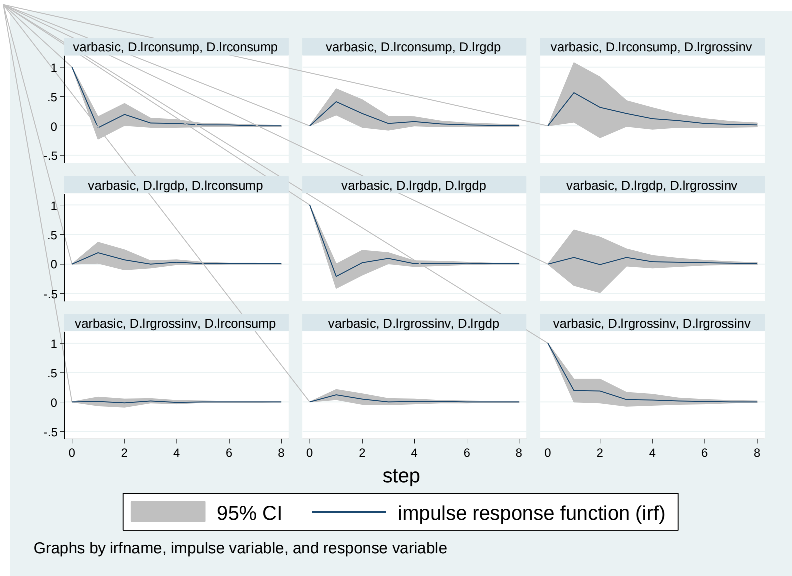


Let us finish this on VARs by looking at a more complex model with one more lag and real GDP also added:

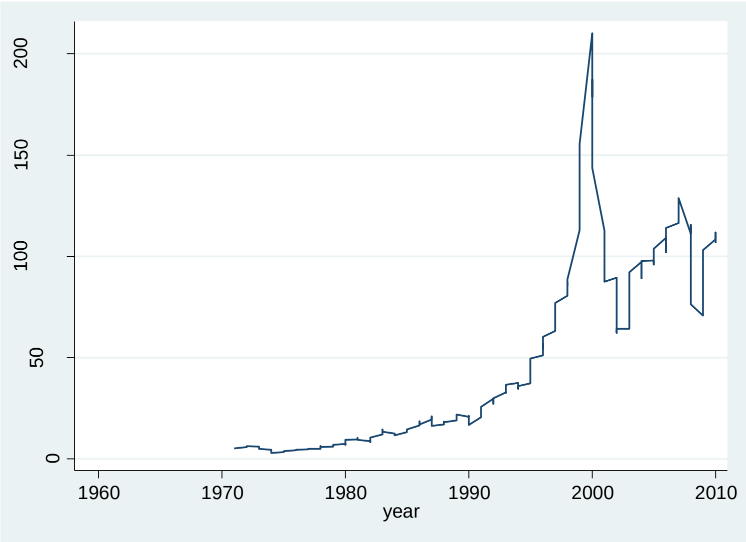
```
. varbasic D.lrgrossinv D.lrconsump D.lrgdp if tin(,2005q4),irf

Vector autoregression

Sample:  1959q4 - 2005q4                        No. of obs      =       185
Log likelihood =  1905.169                      AIC             =  -20.3694
FPE            =  2.86e-13                      HQIC            = -20.22125
Det(Sigma_ml)  =  2.28e-13                      SBIC            = -20.00385
```

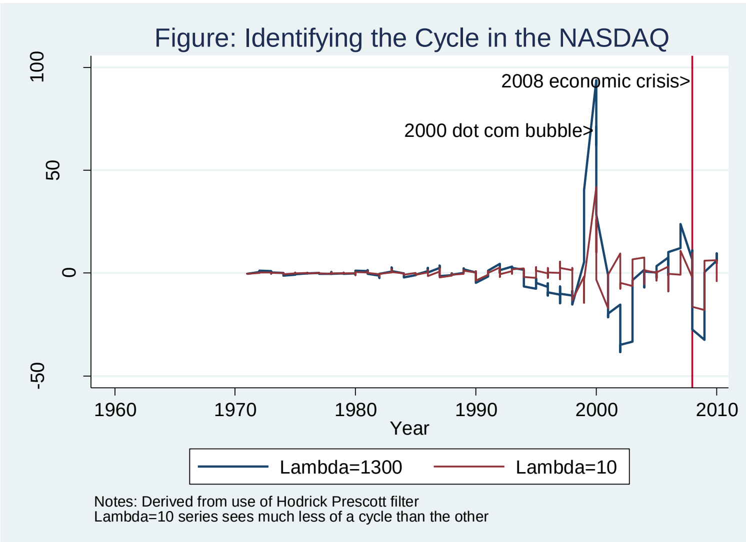
Equation	Parms	RMSE	R-sq	chi2	P>chi2
D_lrgrossinv	7	.017503	0.2030	47.12655	0.0000
D_lrconsump	7	.006579	0.0994	20.42492	0.0023
D_lrgdp	7	.007722	0.2157	50.88832	0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D_lrgrossinv						
lrgrossinv						
LD.	.1948761	.0977977	1.99	0.046	.0031962	.3865561
L2D.	.1271815	.0981167	1.30	0.195	-.0651237	.3194868
lrconsump						
LD.	.5667047	.2556723	2.22	0.027	.0655963	1.067813
L2D.	.1771756	.2567412	0.69	0.490	-.326028	.6803791
lrgdp						
LD.	.1051089	.2399165	0.44	0.661	-.3651189	.5753367
L2D.	-.1210883	.2349968	-0.52	0.606	-.5816736	.3394969
_cons	-.0009508	.0027881	-0.34	0.733	-.0064153	.0045138
D_lrconsump						
lrgrossinv						
LD.	.0106853	.0367601	0.29	0.771	-.0613631	.0827338
L2D.	-.0448372	.03688	-1.22	0.224	-.1171207	.0274463
lrconsump						
LD.	-.0328597	.0961018	-0.34	0.732	-.2212158	.1554964
L2D.	.1113313	.0965036	1.15	0.249	-.0778123	.300475
lrgdp						
LD.	.1887531	.0901796	2.09	0.036	.0120043	.3655018
L2D.	.1113505	.0883304	1.26	0.207	-.0617738	.2844748
_cons	.0058867	.001048	5.62	0.000	.0038326	.0079407
D_lrgdp						
lrgrossinv						
LD.	.1239506	.0431482	2.87	0.004	.0393818	.2085195
L2D.	.043157	.0432889	1.00	0.319	-.0416878	.1280017
lrconsump						
LD.	.4077815	.1128022	3.62	0.000	.1866933	.6288696
L2D.	.2374275	.1132738	2.10	0.036	.0154149	.45944
lrgdp						
LD.	-.2095935	.1058508	-1.98	0.048	-.4170572	-.0021298
L2D.	-.1141997	.1036802	-1.10	0.271	-.3174091	.0890097
_cons	.0038423	.0012301	3.12	0.002	.0014314	.0062533



How to interpret this graph? Take consumption, the impact of a consumption shock on consumption quickly does out, but then recovers again before moving to zero. We can see too the impacts on GDP and investment. Finally we note in the following slides the lecture talks of detrending data. It is easy to fit a nonlinear trend to the data. But as in slide 18 this is often done using the Hodrick Prescott (HP) filter. Many econometric package programs have this as a facility. When using it you have to specify a value of λ. We will illustrate by looking at the NASDAQ index:





Programming Appendix

use http://fmwww.bc.edu/cfb/data/usmacro1

varbasic D.lrgrossinv D.lrconsump if tin(,2005q4) lags(1),irf

varbasic D.lrgrossinv D.lrconsump D.lrgdp if tin(,2005q4),irf

matrix lr = (., 0\0, .)

svar D.lrmbase D.lrgdp, lags(4) lreq(lr) nolog http://fmwww.bc.edu/EC-C/S2013/823/EC823.S2013.nn10.slides.pdf

matrix lr = (.,., 0\0,. ,.\0,.,0)

svar D.lrgrossinv D.lrconsump D.lrgdp if tin(,2005q4) , lags(23) lreq(lr) nolog

tsfilter hp NASDAQ_hp = NASDAQ

tsfilter hp NASDAQ_hp100 = NASDAQindex,sm(100)

line NASDAQ_hp100 year

tsfilter hp NASDAQ_hp1300 = NASDAQindex,sm(1300)

line NASDAQ_hp1300 year

tsfilter hp NASDAQ_hp10 = NASDAQindex,sm(10)

```
label var  trndyr  "Year"
label var  NASDAQ_hp1300  "Lambda=1300"
label var  NASDAQ_hp10  "Lambda=10"
twoway line NASDAQ_hp1300 year, lwidth(medthick) xtitle(Year) ytitle(Filtered series, margin(0 3 0 0)) title(Figure: Identifying the Cycle in the NASDAQ) note("Notes: Derived from use of Hodrick Prescott filter " "Lambda=10 series sees much less of a cycle than the other") xline(2008) text(94 2008 "2008 economic crisis>",place(w)) text(70 2000 "2000 dot com bubble>",place(w)) || line NASDAQ_hp10 year
```