Inflation & the Central Bank

This is based on Blanchard's slides Number 9:

http://ocw.mit.edu/courses/economics/14-452-macroeconomic-theory-ii-spring-2007/lecture-notes/slides09.pdf

The VAR analysis illustrated how macroeconomists do empirical work – not in its entirety but as an example. This lecture illustrates the other side, how macroeconomists build theoretical models from which they draw conclusions. You might say there are two much maths here – but that is what characterises much of modern economics. And the maths is not that difficult, although at times it takes some time to work through the equations.

The Central Bank's Problem

Thus this illustrates how economists write papers and analyse the macroeconomy from a theoretical perspective. A lot of equations, but actually nothing that difficult, just repetitive.

Assume the Phillips curve relationship:

$$y_t = \Upsilon(\pi_t - E_{t-1}\pi_t) \qquad (1)$$

This version is known as the price surprise function. If prices are greater than expected output/employment/ whatever y_t is is higher than normal. If y_t is unemployment then it is lower. The rational is that producers are fooled into thinking the higher prices they are receiving for their goods are higher than others and they expand production. Workers are fooled into thinking their wages are higher in real terms than they are and are prepared to work more than if this was not so.

It can be rewritten as

$$\pi_t = E_{t-1}\pi_t + (1/\Upsilon)y_t \qquad (2)$$

which if y_t is unemployment is a form of Phillips curve. The Central bank minimizes

$$\alpha(y_t - k)^2 + \pi_t^2 \qquad (3)$$

This is a standard 'loss function'. The target for inflation is 0, +2 is as undesirable as -2. Similarly with deviations of y from k, which is the target for y. The best the Bank can do is hit its targets, at this point its loss function equals zero.

Insert (1) into (3):

$$\alpha(\Upsilon(\pi_t - E_{t-1}\pi_t) - k)^2 + \pi_t^2$$

differentiate with respect to π_t in order to find the minimum

$2\alpha(\Upsilon(\pi_t - E_{t-1}\pi_t) - k)\Upsilon + 2\pi_t$

Set=0 [for a minimum] and cancel 2.
$\alpha(\Upsilon(\pi_t - E_{t-1}\pi_t) - k)\Upsilon + \pi_t=0$

$\alpha\Upsilon^2\pi_t - \alpha\Upsilon^2 E_{t-1}\pi_t - \alpha\Upsilon k + \pi_t=0$

$\pi_t + \alpha\Upsilon^2\pi_t = \alpha\Upsilon^2 E_{t-1}\pi_t + \alpha\Upsilon k$ (4)

$\pi_t(1 + \alpha\Upsilon^2) = \alpha\Upsilon^2 E_{t-1}\pi_t + \alpha\Upsilon k$

$\pi_t = (\alpha\Upsilon^2 E_{t-1}\pi_t + \alpha\Upsilon k)/(1 + \alpha\Upsilon^2)$

$\pi_t = (\alpha\Upsilon/(1 + \alpha\Upsilon^2))(\Upsilon E_{t-1}\pi_t + k)$ (5)

At time t-1 assume people's expectations of inflation are correct and $E_{t-1}\pi_t = \pi_t$ [the notes say this is equivalent to rational expectations, seems wrong, but we will still go with. Note too it means $E_t\pi_{t+1} = \pi_{t+1}$]. In this case we have from (4)

$\pi_t + \alpha\Upsilon^2\pi_t = \alpha\Upsilon^2\pi_t + \alpha\Upsilon k$

and thus $\pi_t = \alpha\Upsilon k$ (6)

and from (1) we have $y_t = 0$ (7)

(6) and (7) correspond to conclusion on slide 4. The Bank's optimisation results in y deviating from its target of k and inflation deviating from zero. The slides [No 4] say that this is 'clearly suboptimal', presumably because the Bank misses both the targets. I believe the note give the wrong emphasis to the problem which the slides indicate can be solved by 'greater commitment' to the targets. But to my mind the problem arises because the Bank has two targets which are linked. The Bank can hit the inflation target, but at a cost to output and the Bank cares about both. There is nothing too in these slides about how the Bank is to achieve its targets. In reality it generally has ONE policy instrument, the rate of interest [You might say what about the money supply, until recently two sides of the same coin, back to this later]. Now in general if you have 2 targets you need a minimum of 2 instruments , given that you should be able to hit both targets. SOLUTION, simplify the Bank's objective function, so it cares either about inflation or about output, but not both. Say the bank is to solely to care about inflation. The Government can then focus on output using the instruments it has of taxation and Government spending. Focus on inflation. Inflation increases with α and Υ. A represents the importance the Bank places on the output target in (3). Υ represents the response of y to price surprises. The more responsive …..

Time Consistency

This is a new analysis, beginning slide 5, they called it a NK [(New Keynesian) model because inflation is linked to the output gap] ignore what we have just done and start again.

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} \qquad (8)$$

Let Central Bank minimise:

$$E_t \sum_i [\beta^i(\alpha(x_{t+i} - k)^2 + \pi_{t+i}^2]$$

That is, it is attempting to hit not just targets this period, but also periods into the future. How far into the future is not clear. Apart from that, what the Bank is attempting to maximising is the same as in the previous analysis, apart from the fact that our output variable has changed from y to x [which seems trivial]. In this x is the 'output gap', the gap between output and 'potential output' as defined by the natural rate. The summation is from i=0. β is a discount factor. $\beta<1$ say 0.8. If this is the case it gives a weight of 1 to this periods outcome [$\beta^0=1$], 0.8 to the outcome in t+1 [$\beta^1=0.8$] and 0.64 in t+2] $\beta^2=0.64$] and so on.

Set up the Lagrangean:

$$L = E_t \sum_i [\beta^i(\alpha(x_{t+i} - k)^2 + \pi_{t+i}^2] - 2\delta_1[\pi_t - \lambda x_t + \beta E_t \pi_{t+1}] - 2\delta_2[\pi_{t+1} - \lambda x_{t+1} + \beta E_{t+1}\pi_{t+2}] + \ldots\ldots\ldots\ldots \quad (9)$$

Where $2\delta_2$ and $2\delta_1$ are Lagrangean multipliers [The 2's help in cancelling the algebra later]

Differentiate with respect to X_t [thus i=0]

$$2\beta^0(\alpha(x_t - k) + 2\delta_1\lambda \qquad (10)$$

And with respect to π_t:

$$2\beta^0\pi_t - 2\delta_1 \qquad (11)$$

OK now in these two equations, cancel the 2s and substitute for $\delta_1 = \beta^0\mu_t$ and $\beta^0=1$. Given this:

(10) becomes [when =0 to optimise] $(\alpha(x_t - k) = -\delta_1\lambda = -\mu_t \lambda$ (12)

and for (11):

$\beta^0\pi_t = \delta_1 = \beta^0\mu_t$ which simplifies to $\pi_t = \mu_t$ (13)

Now differentiate L with respect to x_{t+1}

$$2\beta^1(\alpha(x_{t+1} - k) + 2\delta_2\lambda \qquad (14)$$

And with respect to π_{t+1}:

$2\beta^1\pi_t - 2\delta_1 - 2\delta_2$ (15)

[$2\delta_1$ is here because of the rational expectations assumption indicated above that: $E_t\pi_{t+1} = \pi_{t+1}$ In (8). Note too that for π_{t+1} i=1 and hence $\beta^i = \beta^1 = \beta$

OK now in these two equations, cancel 2s and substitute for $\delta_1 = \beta\mu_t$ $\delta_2 = \beta\mu_{t+1}$:

(from 14, setting to 0 to get optimum): $\beta(\alpha(x_{t+1} - k) = -\delta_2\lambda = -\beta\mu_{t+1}\lambda$ which simplifies to $\alpha(x_{t+1} - k) = -\mu_t \lambda$ (16)

and for (15):

$\beta^1\pi_t = \delta_1 + \delta_2 = \beta^0\mu_t + \beta^1\mu_{t+1}$ which simplifies to $\pi_t = \mu_t + \beta\mu_{t+1}$ (17)

On slide 6 these are referred to as first ordered conditions (FOC)
From (12) and (13)

$\alpha(x_t - k) = -\lambda\pi_t$

$\alpha x_t = \alpha k - \lambda\pi_t$

$x_t = k - (\lambda/\alpha)\pi_t$ [Corresponds to eq 1 slide 7] (18)

Replace in the Phillips curve (8) to get:

$\pi_t = \lambda x_t + \beta E_t\pi_{t+1}$ (8)

$\pi_t = \lambda[k - (\lambda/\alpha)\pi_t] + \beta E_t\pi_{t+1}$
$= \lambda k - (\lambda^2/\alpha)\pi_t + \beta E_t\pi_{t+1}$

$\pi_t + (\lambda^2/\alpha)\pi_t = \lambda k + \beta E_t\pi_{t+1}$

$\pi_t[1 + (\lambda^2/\alpha)] = \lambda k + \beta E_t\pi_{t+1}$

$\pi_t = [\lambda k + \beta E_t\pi_{t+1}]/[1 + (\lambda^2/\alpha)]$ [Equivalent to eqn 2 slide 7]

$\pi_t [1 + (\lambda^2/\alpha)] = [\lambda k + \beta E_t\pi_{t+1}]$

$\pi_t [1 + (\lambda^2/\alpha)] - \beta\pi_t = [\lambda k]$

$\pi_t [1 + (\lambda^2/\alpha) - \beta] = [\lambda k]$

$\pi_t = \lambda k/[1 + (\lambda^2/\alpha) - \beta]$

$\pi_t = \lambda k \alpha /[\alpha + \lambda^2 - \beta\alpha]$

$\pi_t = \lambda k \alpha /[\alpha(1-\beta) + \lambda^2]$ [Equivalent to eqn 3 slide 7]

Going back to the Phillips curve:

$\pi_t = \lambda x_t + \beta E_t\pi_{t+1}$ (1A)

and rearranging:

$x_t = \{\pi_t - \beta E_t\pi_{t+1}\}/\lambda$

Constant inflation means $\pi_t = E_t\pi_{t+1}$, hence:

$x_t = \{\pi_t - \beta\pi_t\}/\lambda$

Substitute from inflation above:

$x_t = \{(1-\beta)\, \lambda k \alpha /[\alpha(1-\beta) + \lambda^2]\}/\lambda$

$x_t = \{(1-\beta)k\alpha/[\alpha(1-\beta) + \lambda^2]\}$ [The final equation on slide 7]

So what does this tell us? If $\beta \sim 1$ [β is approximately 1] ,$\pi_t \sim (\alpha/\lambda)k$, xt ~ 0 So in this case we have positive inflation, and zero output gap, again both miss their targets. The larger α, or the lower λ, or the larger k, the larger the inflation. Let us just remind ourselves:

β: Coefficient on expected inflation in Phillips curve (8). We expect this to be 1, people to fully adjust to expected inflation>
α: The weight the Bank gives to hitting the output target in the equation just after (8).
λ: The coefficient on x_t in the Phillips curve. The greater this is the more inflation responds to changes in the output variable.
k: The output target for the Bank.

The presentation then goes on to talk about how we could get lower inflation [slide 9]. Fairly trivial

1. Reputation Repeated game: If central bank "cheats", then revert to time-consistent solution forever. This seems to be saying lets change all the equations and inflation is what the Central Bank says it will be, until the Central Banker 'cheats', people lose faith in the Bank and the previous solution prevails forever.
2. Tough central banker – which the slide equates to lowering α. This is obvious, but also and what is not said we get larger output gaps.
3. Increase the cost of inflation, this is the cost to the economy so do not index link the tax system, thus when inflation goes up taxes automatically increase. How does this figure in the model? Well the weight the Banker gives to inflation is always 1. This implies giving it a higher relative weight

by once more lowering α. The slides talk about this might change γ in the Phillips curve, but not how or why.

As I have said the basic problem is Central Bankers have only one instrument and with one instrument you can only hit one target.

The rest of the lecture proceeds in the same manner. The first parts looks at how to react to oil shocks, based on a production function with just labour and oil. It also distinguishes between first best output and second best. First best is what we could get in a perfect world. Second best is the best we can achieve given imperfections in the economy such as monopoly firms. On slide 25 it begins to talk about interest rates.